# MILLIMICROSECOND PULSE STUDIES

## Part I

## Ionization Processes in High-Speed Thyratrons

DEPARTMENT OF ELECTRICAL ENGINEERING
CARNEGIE INSTITUTE OF TECHNOLOGY
PITTSBURGH, PENNSYLVANIA

# IONIZATION PROCESSES IN HIGH SPEED THYRATRONS

by

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# PREFACE

The work on ionization processes in High Speed Thyratrons is part of a study carried on by the Electrical Engineering Department of the Carnegie Institute of Technology on analytical studies of elements of milli-microsecond pulse systems. Previous work on hydrogen-thyratron pulse generators has brought to light basic differences in ionization phenomena between hydrogen thyrations and other types of thyration. This report is concerned with an experimental and analytic study of these differences. Its text comprises the doctorate thesis of William C. Dean.

1. "The Initial Conduction Interval in High-Speed Thyratrons", J. B. Woodford, and E. M. Williams, Journal of Applied Physics, Vol. 23, No. 7, pp. 722-724, July, 1952.

IONIZATION PROCESSES IN THYRATRONS

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# INTRODUCTION

Thyratrons are three or sometimes four element gas filled tubes designed to switch large amounts of power quickly. They find wide use as switches in pulse forming circuits for radar, industrial control equipment, and other systems. For most applications the ionization time of these tubes is shorter than necessary and can be considered instantaneous. Methods of slowing down the pulse rise time have been employed to reduce the peak power the thyratrons must handle in an effort to prolong their life. Some new applications, however, require high power pulses with very short rise times. The limits of radar resolution are dependent upon the sharpness of the transmitted pulse. Short, high power pulses are used for example in beam deflection systems for cyclotrons.

In all thyratrons the grid so completely shields the plate from the cathode that even with a large positive plate potential applied conduction does not take place until it is initiated by the grid. When the potential on the grid of the thyratron is raised sufficiently, a small electron current flows to the grid and the positive anode. This space charge limited current causes ionization of neutral gas molecules by electron collision, each ionizing collision forming a new free electron and a positive ion. As more and more positive ions are produced, they tend to neutralize the space charge caused by electrons permitting more electrons and hence more current to flow. In steady state conduction a plasma is established. The mechanism of a plasma is similar to the condition of the electron gas in a metal where a low field intensity gives rise to a large electron current flowing through the positive metallic ions. One main difference is that the positive ions in the metal are fixed in position by the lattice structure while the positive ions in a gas drift in the direction of the electric field. In full conduction the voltage drop across the thyratron is low so that with the plasma established the grid and plate currents in most applications are primarily determined by the external circuitry.

A thyratron may be used in many different circuits, but most of these

can be reduced to the equivalent circuit shown in figure 1.

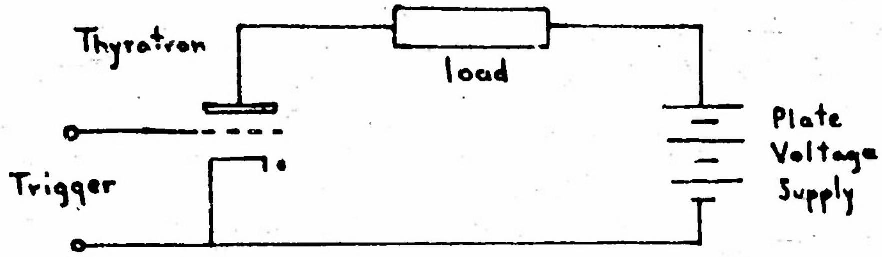



Figure 1.

The ionization interval for a thyratron in such a circuit can be readily divided into two parts. The first part, defined in this thesis as the delay time, is the time interval between the application of the trigger pulse to the grid and the instant when the plate voltage starts to fall rapidly. During the delay interval the plate voltage of the tube does not change appreciably. The second part, defined in this part as the commutation time, is the time interval during which the plate voltage starts to fall rapidly until steady state full conduction is reached. The total ionization time is the sum of the delay time and the commutation time and is the time required to form the plasma.

For thyratrons used in pulse forming circuits, the commutation time of the thyratron defines the shortest pulse rise time available. The study of the dependence of the commutation time on gas pressure and tube geometry is the primary purpose of this thesis, but, in addition, a study of the delay time must also be made in order to understand the commutation interval.

REVIEW OF OTHER WORK IN THE FIELD

Considerable attention has been given in recent years to a study of the deionization time in thyratrons,[6,7,8] since this limits the maximum repetition rate of the tube. Less attention, however, has been given to the ionization time since conduction is fast enough for most applications.

As early as 1933 Snoddy[2] investigated the delay time of mercury thyratrons. The wave shape of the anode voltage fall was attributed to the effect of the external circuitry and interelectrode capacities of the tube.

In 1933 Wheatcroft[3] did some analytical work on the currents during the delay time based on Dushman's equation[7], namely $I = A\epsilon^{-\frac{E}{kT}}$

where T is the emission temperature of the cathode in degrees Kelvin

k is the Boltzmann constant

E is the electron energy required to pass the work function barrier of the cathode surface

A is an empirical constant

Wheatcroft[3], working with a type "BT 1" thyratron whose grid is a circular disk with a concentric hole 1/40 th. the grid area, makes the plane electrode assumption and states that the electrons having sufficient energy to get past the potential well caused by space charge in the grid cathode region after the grid is triggered give rise to a cathode current of

$$I_c = I_s \epsilon^{-\frac{V_m q}{kT}}$$

provided the hole is so large that all electrons with sufficient energy got into the plate region. If not, the relationship becomes

$$I_c = \eta I_s \epsilon^{-\frac{V_m q}{kT}}$$

where $\eta$ is a geometrical constant based on the grid hole size.

$q$ is the electronic charge

$V_m$ is the potential of the well below that of the cathode

$I_c$ is the cathode current

$I_s$ is the total emission of the cathode

Ionizations which occur in the grid-anode region change the effect of the anode potential in the cathode region and therefore increase the anode current. At breakdown Wheatcroft states that $\frac{dV}{dI} \rightarrow \infty$

A similar approach yielding the anode current rise during both the delay and the commutation intervals is given by Mullin[4]. His explanation of delay time is based on the rate of ionization in the grid-anode region and

the rate at which the positive ions move back into the potential well neutralizing it. When the potential of the well has been reduced to zero, Mullin concludes that the cathode is emitting its maximum current, and the initiation of the discharge is complete.

In 1940 Harrison[5] experimentally investigated the delay time of argon, neon, and mercury thyratrons. The delay time was found to be inversely proportional to the grid overvoltage. Increasing the anode voltage also shortened the delay time. No theories explaining these findings were proposed.

In 1950 Woodford[1] measured the variation of breakdown time on 5C22 hydrogen thyratrons with anode voltage, grid overvoltage load impedance, and lead inductance. The 5C22 thyratrons and similar types differ considerably in geometry from the mercury and other types with regard to the shielding of the plate by the grid.

To date the analytical work on thyratrons has been confined to tubes with the geometry characteristic of the mercury thyratron. Furthermore, no experimental data or analytical treatment of the variation with pressure of either the delay time or the commutation time has been given in the literature. In the first section of this thesis the very much shorter pulse rise times available from the 5C22 hydrogen thyratron as compared to those of the mercury thyratron are shown to result largely from its special geometric construction. The second section of this thesis describes a new approximate analysis of the effect of pressure on the commutation time of hydrogen thyratrons and compares the analytical results with experimentally determined values. An operational criterion for obtaining the minimum commutation time when using hydrogen thyratrons similar to the 5C22 is also given.

GEOMETRICAL DIFFERENCES IN THYRATRONS

Hydrogen thyratrons, such as the 3C45, 4C35, and 5C22 thyratrons, developed during the last war differ significantly in electrode geometry from industrial mercury vapor thyratrons. In the mercury thyratron a direct line-of-sight path for electron current flow exists between the cathode and the plate through a concentric hole in a disk-shaped grid. The plate current is cutoff only by the potential barrier arising when one or more of the grids are held at a negative potential. On the other hand, the grid and grid baffles in the hydrogen thyratron so completely enclose the plate from the cathode that the grid acts as a physical as well as a potential barrier for electron plate current. This shielding in the hydrogen thyratrons, in addition to permitting higher anode working voltages, causes operational differences in the ionization processes. For the purposes of this comparison the hydrogen thyratron construction just described will be known as the highly shielded type, and the mercury thyratron type of construction as the line-of-sight type. Of course, the highly shielded type of construction is not necessarily limited to hydrogen-filled tubes nor are the line-of-sight tubes only mercury-filled. For a comparison of the two types of construction see figure 2. This section describes the effect of these two geometries on the delay and commutation times.

In the line-of-sight tubes some of the initial electrons pulled toward the grid by the grid overvoltage pass through the grid hole into the grid anode region and on the the anode. Thus plate current flows as soon as space charge limited current is set up in the tube. Ionizations can then take place in the grid-anode region with the plate current increasing as the space charge is neutralized and the plasma is formed. This mechanism in the line-of-sight tubes has been handled analytically by Wheatcroft[3] and by Mullin[4]. Wheatcroft described the cathode current $I_c$ as a function of the grid overvoltage $V_g$ times the triode amplification factor $\mu$ plus the anode voltage $V_a$;

# Element Structures
## of the
## 5C22 Hydrogen Thyratron
## and the
## FG-172 Mercury Thyratron

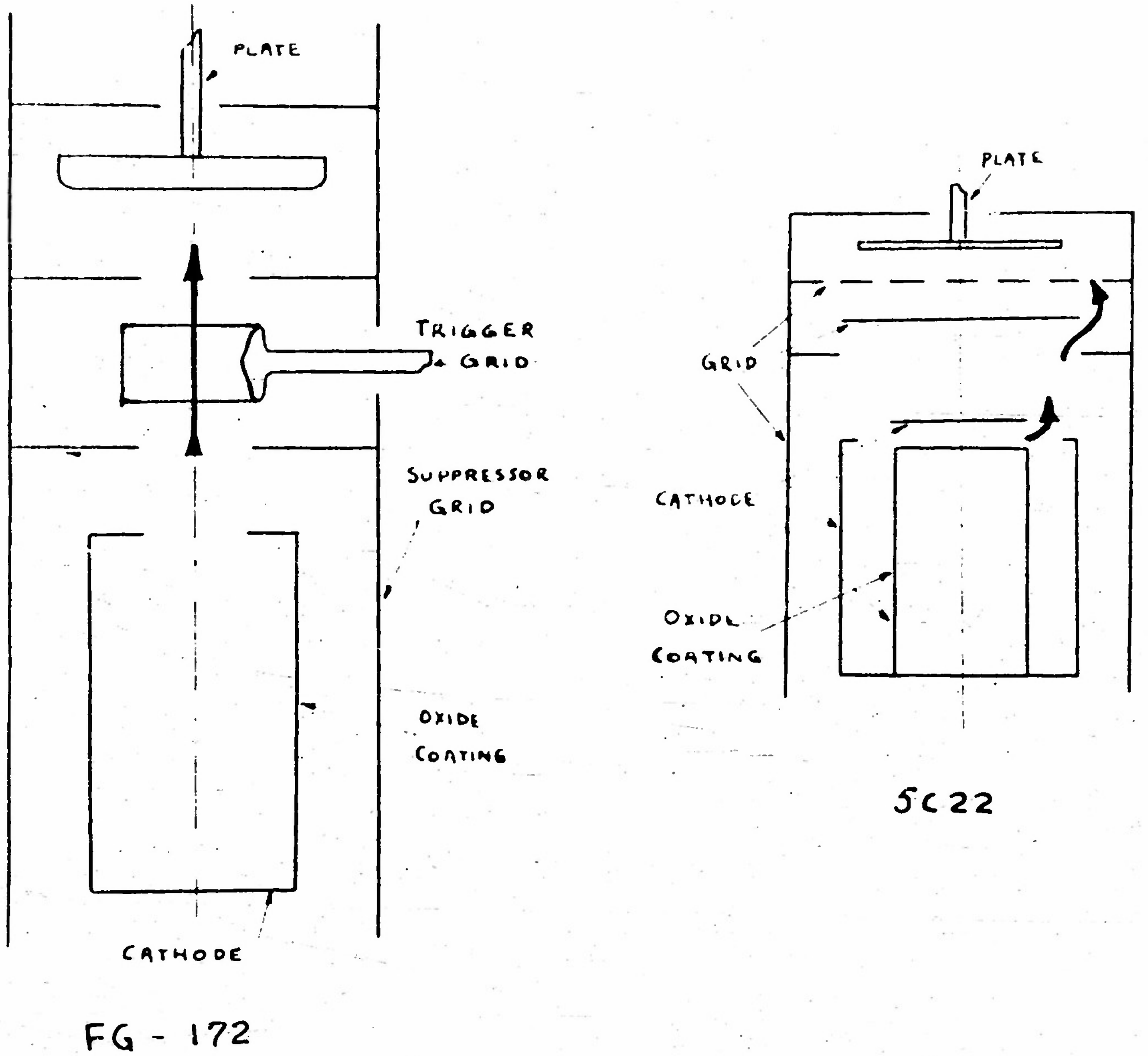



Figure 2: The heavy arrows indicate the path of electron plate current.

that is, the grid voltage volt for volt has $\mu$ times as much effect as the anode voltage in determining the cathode current during the delay interval. This effect is the same as the effect of the grid voltage in a normal triode at all times.

$$I_c = \text{fcn.}(\mu V_g + V_a)$$

Wheatcroft[3] and Harrison[5] found the delay time in line-of-sight thyratrons to decrease with increasing anode voltage. Mullin found the delay time relatively independent of the anode voltage showing that experimental differences between investigators existed.

In the highly-shielded hydrogen thyratrons, on the other hand, initial electrons cannot find their way into the grid-anode region. Instead, a two-step process is required to initiate the discharge. In the first step, a plasma is formed in the grid-cathode space under the action of the positive grid. In the second, an appreciable number of electrons from the grid-cathode plasma find their way around the grid baffles to initiate a plasma in the grid anode region, completing the formation of the discharge. During the first of these two steps, essentially the delay period, the cathode current is independent of anode voltage, and the cathode current is flowing almost entirely in the grid circuit.

$$I_c = \text{fcn.}(V_g) \neq \text{fcn.}(V_a)$$

The distinction between the two types of thyratrons is demonstrated by the photographs of grid voltage and plate voltage waveforms for both a highly shielded hydrogen tube and a line-of-sight mercury tube.

The grid voltage waveform shown was measured between the grid and cathode terminals of the thyratron. The pulse generator supplying this grid voltage maintains a high grid over-voltage until the grid cathode region ionizes. Then, because of the voltage drop caused by the flow of grid current

in the internal impedance of the pulse generator, the grid voltage is reduced to a value near the ionisation potential of the gas. This is shown in Figure 3, where the first drop in grid voltage occurs when the grid ionises and the second at the termination of the triggering pulse.

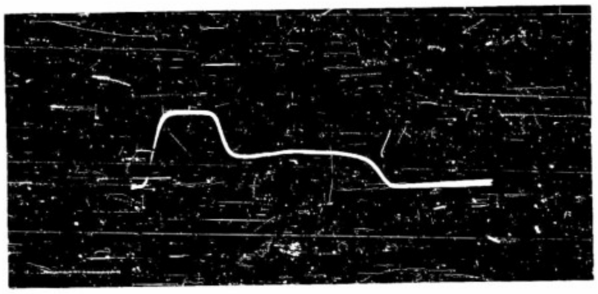


Figure 3.

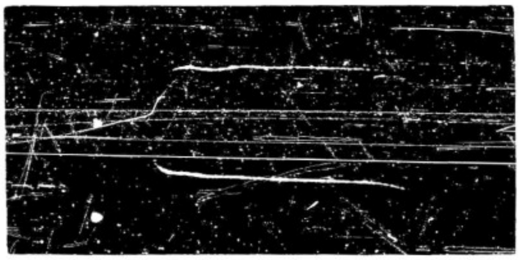
The waveform shown is with zero plate voltage. The photographs of figure 4 on the next page show that with zero plate voltage the grid voltage waveforms of the highly-shielded hydrogen tubes and the line-of-sight mercury tubes are very similar. However, the plate voltage drop can occur in mercury tubes before the grid-cathode region ionises completely, whereas in the hydrogen tubes the grid-cathode region must be ionised first. This effect is shown by the fact that if the trigger pulse is shortened to less than the time required for the grid-cathode region to ionise, the line-of-sight tube can still be made to fire with sufficient anode voltage, but the highly shielded hydrogen tube cannot.

The effect upon commutation time of the geometry differences in the geometry of the two types of thyratrons is explained as follows: In the line-of-sight tubes positive ions must neutralise the space charge in the cathode-grid region before a large plate current can flow. The time required for this neutralisation is calculated by Mullin[4]. The neutralisation of this space charge is a gradual process, and as it progresses, the plate current increases gradually. During the delay time the dominant field drawing electrons into

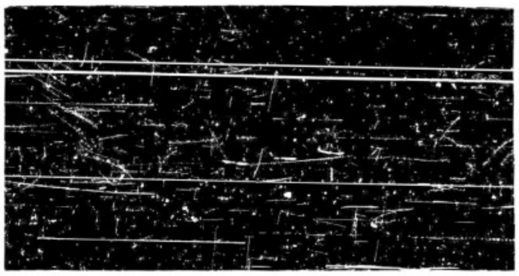
Thyratron Grid Current, Grid Voltage, and Plate Volt age Waveforms.

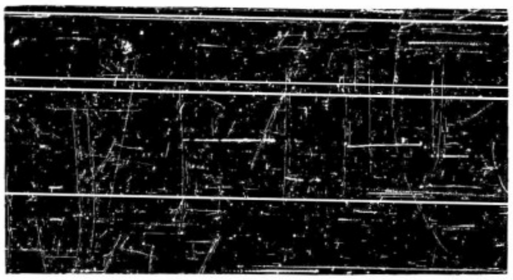
| FG-172 Mercury Line-of-sight Thyratron | 5C22 Hydrogen Highly-shielded thyratron |
|---|---|

The upper traces are grid current, the bottom traces grid voltage, with zero plate voltage applied. Here the waveforms of the tubes are quite similar. When the grid ionizes, grid current rises sharply and grid voltage falls. The initial small pulse of grid current on the hydrogen trace is due to the charging of the grid-to-cathode capacity. Because of the very small grid-to-cathode capacity in the FG-172, a similar pulse does not appear on the grid current trace of the mercury tube.



The same traces are shown here with 750 volts on the plate of each tube. The effect of plate voltage on the mercury tube is only to shorten the delay time until the grid ionizes indicating that the ionizations in the plate-grid region and the cathode-grid region occur concurrently. In the hydrogen thyratron as the plate-grid region ionizes, a positive potential spike and a negative current spike appear on the grid as the grid tries to rise to the plate potential. This grid spike cannot be made to occur ahead of the time when the cathode-grid region just starts to ionize, and the grid delay time is not shortened. Hence, the highly shielded tube must fire in a two step process: the cathode-grid region ionizes first and the plate-grid region second. All trace lengths are five microseconds.





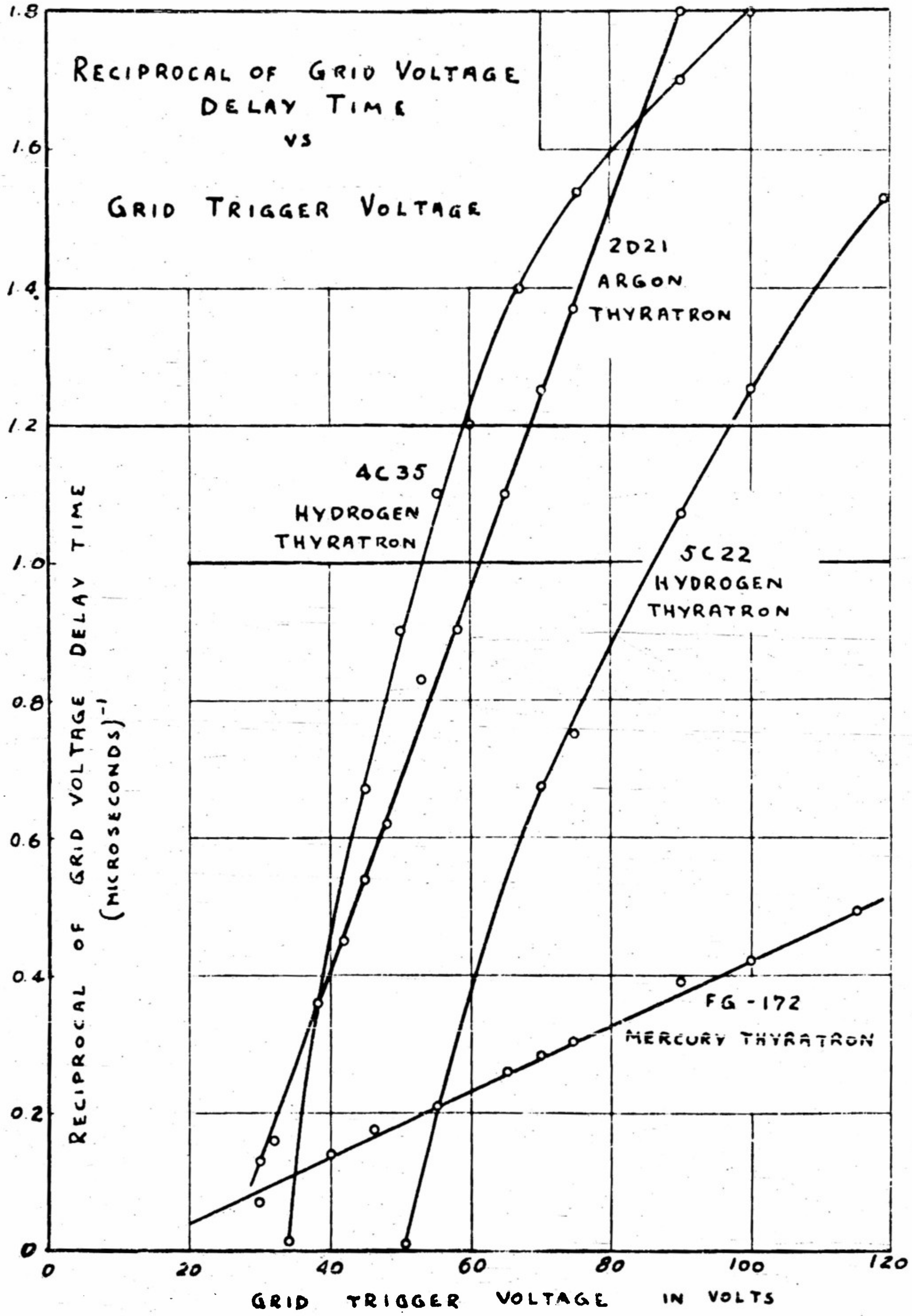
The above traces show plate voltage to the same time scale as the grid current and grid voltage traces. The repeated pulses are negative reflections from the end of the transmission line plate load.

Figure 4

the plate-grid region is due to the grid voltage, since the plate voltage has so little effect in the cathode-grid region.

In the highly shielded tubes positive ions must neutralize the space charges in the cathode-grid region also, but no plate current flows until the cathode-grid plasma is established. Then when the plate-grid region starts to breakdown, the source of electrons is not a thermionic cathode with a space charge to neutralize but rather a plasma of positive ions and electrons near the grid. The field drawing electrons into the plate-grid region is the high anode field itself rather than the lower cathode-grid field. Consequently, the highly-shielded type of construction results in much shorter commutation times than the line-of-sight type.

The thyratrons of the highly-shielded geometry also exhibit different characteristics with regard to delay time versus grid-overvoltage. If the delay time of both the highly shielded tubes and the line-of-sight tubes is measured as the time to ionize the grid-cathode region with zero or low values of plate voltage, then the delay time is inversely proportional to the grid overvoltage for the line-of-sight tubes. Both Harrison[5] and Mullin[4] verified this relation for the line-of-sight tubes. However for the highly shielded hydrogen tubes this function of grid overvoltage is non-linear. The curves on the following page of graph 1 illustrate this result. Besides the plate of the hydrogen tubes being highly shielded, the cathode is shielded by baffles at cathode potential to prevent positive ion bombardment of the oxide coating. It is possible that this non-linearity is caused by the extra baffles in the highly shielded thyratron which the line-of-sight tubes do not have.


RECIPROCAL OF GRID VOLTAGE DELAY TIME
VS
GRID TRIGGER VOLTAGE
2D21
ARGON
THYRATRON
4C35
HYDROGEN
THYRATRON
5C22
HYDROGEN
THYRATRON
FG-172
MERCURY THYRATRON
RECIPROCAL OF GRID VOLTAGE DELAY TIME
(MICROSECONDS)$^{-1}$
GRID TRIGGER VOLTAGE IN VOLTS
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
0
20
40
60
80
100
120

GRAPH 1.

## EFFECTS OF PRESSURE

Since the gas with which a thyratron is filled permits the tube to carry its characteristically high currents compared with the currents in a high vacuum tube, variation of the gas pressure would be expected to have considerable effect upon the tube operation. This section describes the effects of gas pressure upon the ionization processes of the thyratron.

In order for an electron to ionize a gas molecule by collision it must have sufficient energy, that is, it must fall freely through a sufficiently large potential difference. With a given field at very low pressures the electrons may fall through a potential difference large enough to give it the energy required for an ionizing collision, but few collisions take place due to the low gas density so that the time rate of ionization is low. In this instance the ionization rate would be increased by increasing the gas pressure. At higher pressures, on the other hand, the gas density may become so great that most electrons will colide with gas molecules before they fall through a sufficiently large potential difference. In this case, increasing the gas pressure further reduces the free paths of the electons, reducing their energies upon collisions, and so decreases the ionization rate. These two effects work against each other so that the maximum ionization rate should be obtained somewhere between these low and high pressure extremes. With a few simplifying assumptions these two effects can be illustrated analytically.

Assume that a uniform fixed field exists across a one centimeter gap filled with hydrogen at a variable pressure. Assume also that one electrode is a thermionic cathode with ample electron emission and that the initial velocities of the electrons are zero. Assume that on the average the net forward velocity of all electrons after collision is zero.

Let the total number of electrons at any time flowing toward the positive electrode be $n = n(t)$

The number of electrons at a time $t$ whose free path is larger than the distance x is[9] $$n\epsilon^{-\frac{x}{\ell}}$$

The mean free path $\ell$ is inversely proportional to pressure so the pressure dependence may be added.

$$n\epsilon^{-\frac{xp}{\ell p_0}}$$

Here $p_0$ is some standard pressure, at which $\ell$ was determined.

The field is E, the voltage across the gap is V, and the electrode separation is D;

$$E = \frac{V}{D}$$

The ionising ability of electrons is a function of the electron energy.

If $\alpha$ is defined as the ions formed per electron per centimeter of path traveled by the electron, then to a good approximation for electron energies of 100 volts and less;[10]

$$\alpha = Kp(Ex - V_i)$$

where K is a constant depending upon the gas,

E is the electric field.

x is the distance traveled by the electron.

$V_i$ is the ionising potential of the gas, a constant

P is the gas pressure. $\alpha$ is proportional to pressure because the number of collisions per centimeter for a given electron velocity is proportional to the gas density.

Curves of $\alpha$ versus electron energy are shown in von Engel and Steenbeck,[11] for several gases.

The quantity B is defined as

$$\beta = \alpha v$$

where B = ions formed per electron per second

and v = average velocity of colliding electrons in centimeters per second.

In a uniform field the force F on an electron of charge q is

$$F = qE = ma$$

so that the acceleration (a) is constant.

Therefore the velocity is a linear function of time and the average velocity of the electron between collisions is half its final velocity because it was assumed that the initial electron velocities are zero.

$$v_{ave.} = v = \frac{1}{2} v_{final}$$

In these calculations, the very small relativistic effects at potentials on the order of 100 electron volts and less are negligible.

The final velocity is determined by the potential difference through which the electron falls.

$$\text{Kinetic energy} = \frac{1}{2} m v_{final}^2 = qEx$$

$$v_{final} = \left( \frac{2qEx}{m} \right)^{1/2}$$

$$v = \frac{1}{2} \left( \frac{2qEx}{m} \right)^{1/2} = \left( \frac{qEx}{2m} \right)^{1/2}$$

Then $\beta = \kappa p \left( Ex - V_i \right) \left( \frac{qEx}{2m} \right)^{1/2}$ $\frac{\text{ions formed}}{\text{electron second}}$

with the restriction $\left( Ex - V_i \right) \geq 0$ or $\beta \geq 0$ since negative ionizations have no meaning.

Then

$n(t)$ = number of electrons flowing at time (t)

$n(t + \Delta t)$ = number of electrons flowing at time $(t + \Delta t)$

$n(t+\Delta t) - n(t)$ = increase in the number of electrons in $\Delta t$ seconds.

The quantity B varies for electrons of different energies, that is, for electrons of different free paths. But B is a function of X, so:

$n(t)\, e^{-\frac{x p}{\ell p_0}}$ = number of electrons whose free paths are greater than X.

$n(t)\, e^{-\frac{(x+\Delta x) p}{\ell p_0}}$ = number of electrons whose free paths are greater than X + ΔX

Then $n(t)\left[\epsilon^{-\frac{x p}{\ell p_0}}\left(1-\epsilon^{-\frac{\Delta x p}{\ell p_0}}\right)\right]$ = number of electrons whose free paths are between $X$ and $X+\Delta X$

Expanding $\left(1-\epsilon^{-\frac{\Delta x p}{\ell p_0}}\right)$ in a series,

$$1-\epsilon^{-\frac{\Delta x p}{\ell p_0}} = 1-1+\frac{1}{1!}\left(\frac{\Delta x p}{\ell p_0}\right)-\frac{1}{2!}\left(\frac{\Delta x p}{\ell p_0}\right)^2+\frac{1}{3!}\left(\frac{\Delta x p}{\ell p_0}\right)^3-$$

$$1-\epsilon^{-\frac{\Delta x p}{\ell p_0}} = \frac{\Delta x p}{\ell p_0}$$ neglecting higher order terms.

The increase in electrons (or ions) is the number of electrons with sufficient energy to ionize times their ionizing effect per second times the number of seconds they act $\Delta t$.

So:

$$n(t+\Delta t)-n(t) = \int_{x=\frac{V_i}{E}}^{x=D} \beta\, n(t)\, \epsilon^{-\frac{x p}{\ell p_0}} \frac{p}{\ell p_0}\, dx\, \Delta t$$

The upper limit is $x = D$ since electrons with free paths greater than D will strike the second electrode and hence not contribute to the ionization. Secondary emission is not considered.

$$\lim_{\Delta t\to 0} \frac{n(t+\Delta t)-n(t)}{\Delta t} = n(t)\int_{x=\frac{V_i}{E}}^{x=D} \beta\, \epsilon^{-\frac{x p}{\ell p_0}} \frac{p}{\ell p_0}\, dx$$

$$\frac{dn}{dt} = n\int_{x=\frac{V_i}{E}}^{x=D} \kappa p\,(Ex-V_i)\left(\frac{qEx}{2m}\right)^{1/2} \epsilon^{-\frac{x p}{\ell p_0}} \frac{p}{\ell p_0}\, dx$$

The expression under the integral sign is now independent of n and t, but is a function of the pressure p and of the applied voltage since the voltage determines the uniform field E.

Let

$$C = \int_{x=\frac{V_i}{E}}^{x=D} \kappa p\,(Ex-V_i)\left(\frac{qEx}{2m}\right)^{1/2} \epsilon^{-\frac{x p}{\ell p_0}} \frac{p}{\ell p_0}\, dx$$

and C will be known as the ionization rate.

The conditions in triggering a 5C22 hydrogen thyratron will now be simulated with regard to gas pressure, trigger voltage on the grid, and grid-cathode separation to shown the correlation between the delay time of a hydrogen thyratron and pressure.

$$C = \frac{\kappa p^2}{\ell p_0} E^{3/2} \left(\frac{q}{2m}\right)^{1/2} \int_{x=\frac{V_i}{E}}^{x=D} \left(x - \frac{V_i}{E}\right) x^{1/2} \epsilon^{-\frac{\alpha p}{\ell p_0}} dx$$

Letting $\kappa E^{3/2} \left(\frac{q}{2m}\right)^{1/2} = A$

and $\frac{V_i}{E} = \bar{X}$

$$C = A \frac{p^2}{\ell p_0} \int_{x=\bar{X}}^{x=D} (x - \bar{X}) x^{1/2} \epsilon^{-\frac{\alpha p}{\ell p_0}} dx$$

The details of the integration of the ionization rate C are carried out in the appendix, and the ionization rate is plotted as a function of pressure for two different values of trigger voltage on the 5C22 grid. These curves are compared with experimental curves of grid voltage delay times versus pressure on graph 2.

According to the theory just presented the ionization rate is fixed at constant voltage and pressure. It might appear therefore that the current buildup in the grid circuit as a function of time could be found by integrating the differential equation

$$\frac{dn}{dt} = Cn$$

and applying Ohm's law to the external circuit. However the solution would be quite limited for no account has been made of ionization by secondary emission, no consideration of any loss of ions by diffusion and other mechanisms, and the field is not linear as assumed but varies in time as the current builds up.

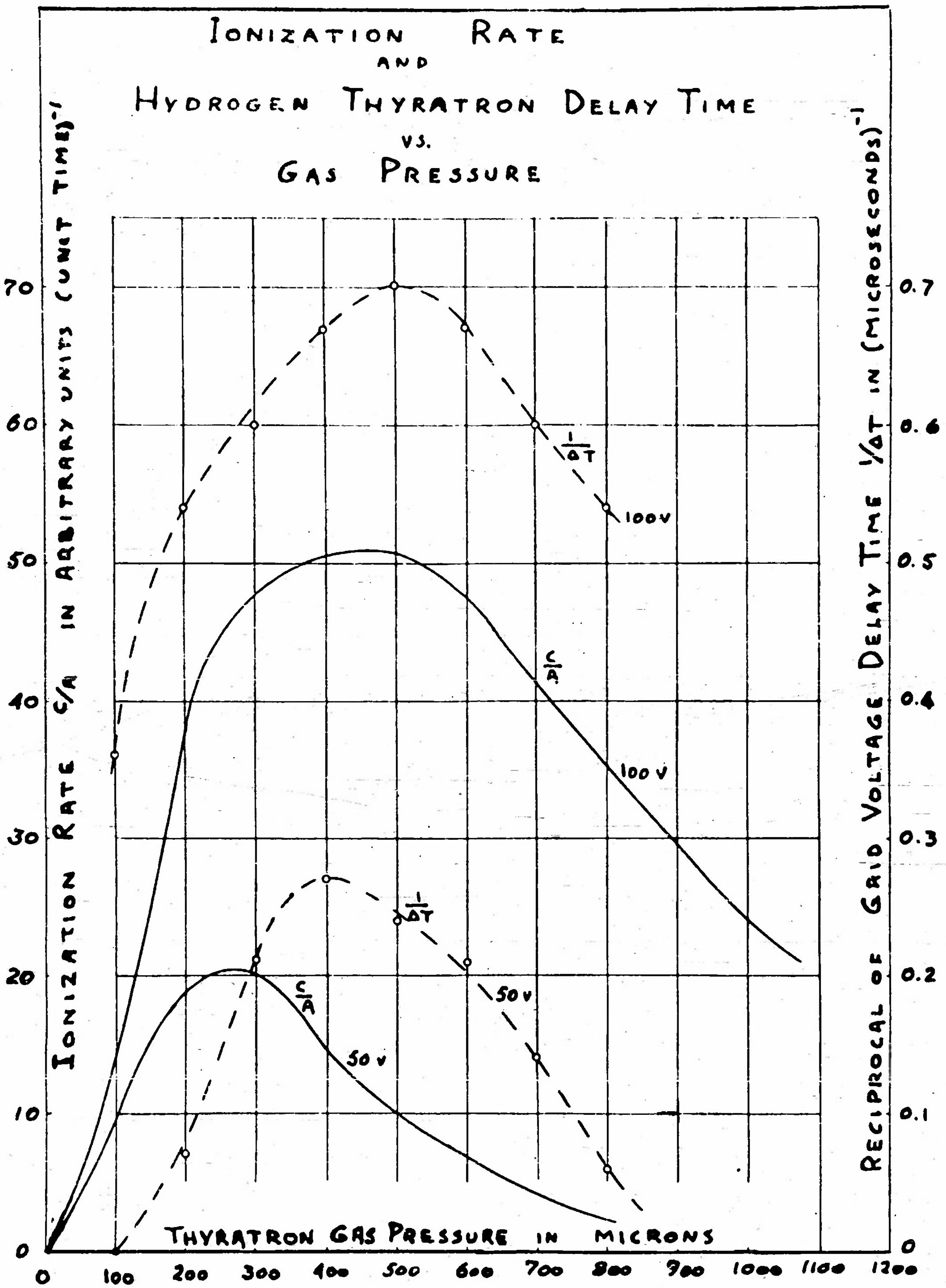
Nevertheless, for any sizeable current increase over the space charge

limited current a plasma must be formed. The time rate of formation of the plasma will depend upon the rate of production of positive ions. Hence the grid voltage delay time of a thyratron should decrease as the rate of ionization increases, and if the governing mechanism in the tube is ionization by electron collision, the delay time should be nearly inversely proportional to the calculated ionization rate.

An experimental check on this theory was made using a special 5C22 hydrogen thyratron equipped with a hydrogen reservoir built by Kuthe Laboratories.[12] The hydrogen reservoir emits or absorbs hydrogen depending upon the temperature of an enclosed filament. Similar reservoirs are added to the larger hydrogen thyratrons to compensate for gas cleanup by the metal and glass surfaces with age. In this particular special 5C22 the tube pressure was calibrated with reservoir filament voltage against a McCleod gauge before the tube was sealed. Due to gas cleanup with age the calibration will not remain fixed, but the relative calibration should remain the same, and all of the tests recorded in this paper were done before the tube had 100 hours of operation.

Pressure dependent curves comparing the analytical ionization rate and the experimentally measured grid voltage delay time are shown on the next page. The important point of these curves is the fact that they possess one maximum in operational pressure range of the hydrogen thyratron. The absolute value of the ionization rate in ion pairs formed per second is not important since the ionization rate is not used to derive the current waveform in the grid circuit in this paper. Consequently the ionization rate is plotted in arbitrary units, the scale being a constant times the true ionization rate. For ease of comparing the analytical and experimental curves the reciprocal of delay time is plotted giving two curves with maximums. The maximum ionization rate and the shortest delay occur at nearly the same pressure. Further, for the higher grid trigger voltage, the maximum ionization rate and minimum delay

occur at a higher pressure, and the ionisation rate is larger and the delay shorter for all pressures. This result is to be expected since higher trigger voltages give shorter delays, as is shown by the curves on graph 2 for several different kinds of thyratrons.


IONIZATION RATE AND HYDROGEN THYRATRON DELAY TIME VS. GAS PRESSURE
IONIZATION RATE C/A IN ARBITRARY UNITS (UNIT TIME)$^{-1}$
RECIPROCAL OF GRID VOLTAGE DELAY TIME 1/ΔT IN (MICROSECONDS)$^{-1}$
THYRATRON GAS PRESSURE IN MICRONS
$\frac{1}{\Delta T}$ 100 V
$\frac{C}{A}$ 100 V
$\frac{1}{\Delta T}$ 50 V
$\frac{C}{A}$ 50 V
0 10 20 30 40 50 60 70
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7
0 100 200 300 400 500 600 700 800 900 1000 1100 1200

GRAPH 2.

## LIMITATIONS OF ANALYSIS

The maxima of the analytical and experimental curves do not occur at just the same pressure for a number of reasons. First of all, the mean free path ( $\ell$ ) of an electron in a gas is dependent upon the temperature of the gas[13] as follows:

$$\ell = \frac{\ell_0 p_0}{p}\left(\frac{T_{gas}}{273}\right)$$

Where $\ell_0$ is the mean free path at a pressure $p_0$ and a temperature of the gas, $T_{gas}$, of 273 degrees Kelvin. The true temperature of the gas in the 5C22 special thyratron is unknown and must be approximated. Secondly, the electron free path is a function of electron velocity.[13] Hence the number of electrons having free paths greater than x is something different than the assumed number $n(t)\,\epsilon^{-\frac{x}{\ell}}$ . Finally, the net forward velocity of all electrons after collision is probably not zero as assumed but rather some small component in the direction of the field.

With the major objective to learn the limitations of the plate voltage pulse, perhaps the more interesting problem would be to apply the analysis for ionization rate to the plate circuit in order to predict the pressure of minimum plate commutation time. This problem is more difficult and less accurate than the grid cathode breakdown because of the higher voltages involved. The assumption that the ions formed per centimeter path per electron, $\alpha$ , is proportional to the potential the electron has upon collision less the ionization potential of the gas ( $V - V_i$ ) is very poor for electron energies over 100 volts. The curves of $\alpha$ for most gases reach a peak near 100 to 150 volts and fall off for higher electron energies[11]. Some other than linear functional approximation or a graphical integration would be required. Further the assumption that the free path of an electron in the gas is not a function of electron velocity is less valid because of the wider range of electron velo-

cities. And last of all the assumption that the net forward velocity of all electrons after collision equals zero is less valid since the many high energy electrons loose only a small fraction of their total energy upon collision. To be sure other approximations than these could be made, but at the higher voltages just what assumptions to make becomes more problematical.

COMMUTATION TIME VERSUS PRESSURE

In spite of the limitations of the analytic work, the ionisation rate theory can be used to explain qualitatively the changes in commutation time with pressure. This section first considers the particular problem of obtaining the shortest possible commutation time in a tube with fixed geometry, operating at a fixed anode supply voltage, with the gas pressure variable. The manner in which the pressure must be varied to obtain the shortest commutation time as the voltage is increased is then investigated.

At moderately low anode voltages (less than 2000 volts for the thyratron used in this investigation) a clear minimum commutation time can be obtained as pressure is varied. Graph 3 indicates such a minimum, obtained with the variable pressure thyratron with an anode supply voltage of 1750 volts. This minimum occurs when the ionisation rate has a maximum value. The ionisation rate in hydrogen is a maximum when most of the electrons have energies between 50 and 200 electron volts upon collision with gas molecules.[11] Since the electron energies upon collision must be maintained in this range for maximum ionisation rate for any condition of tube operation, then changes in anode supply voltage must be accompanied by proportional changes in pressure. For example, if there is an increase in the anode-grid field, the free paths of the electrons must be reduced proportionally in order that the electron energy distribution upon collision remains at the same optimum value. The anode-grid region in the highly shielded construction is well approximated by two parallel plate electrodes separated by a distance d. For this geometry, the conditions for maintaining optimum ionisation rate can be written as

$$E = \frac{V}{d} = k_o p$$

where $k_o$ is an empirical constant. This follows because the free paths will be inversely proportional to the pressure.

Then, $$V = k_o pd$$

This indicates that to obtain a minimum commutation time, the pressure must be increased proportionally as the voltage is increased. By keeping pressure proportional to the field, the free paths will be inversely proportional to the field, the free paths will be inversely proportional to the field and the energy distributions of the electrons will be essentially unchanged.

Hence $$E = \frac{V}{d} = k_o p$$

and $$V = k_o pd$$

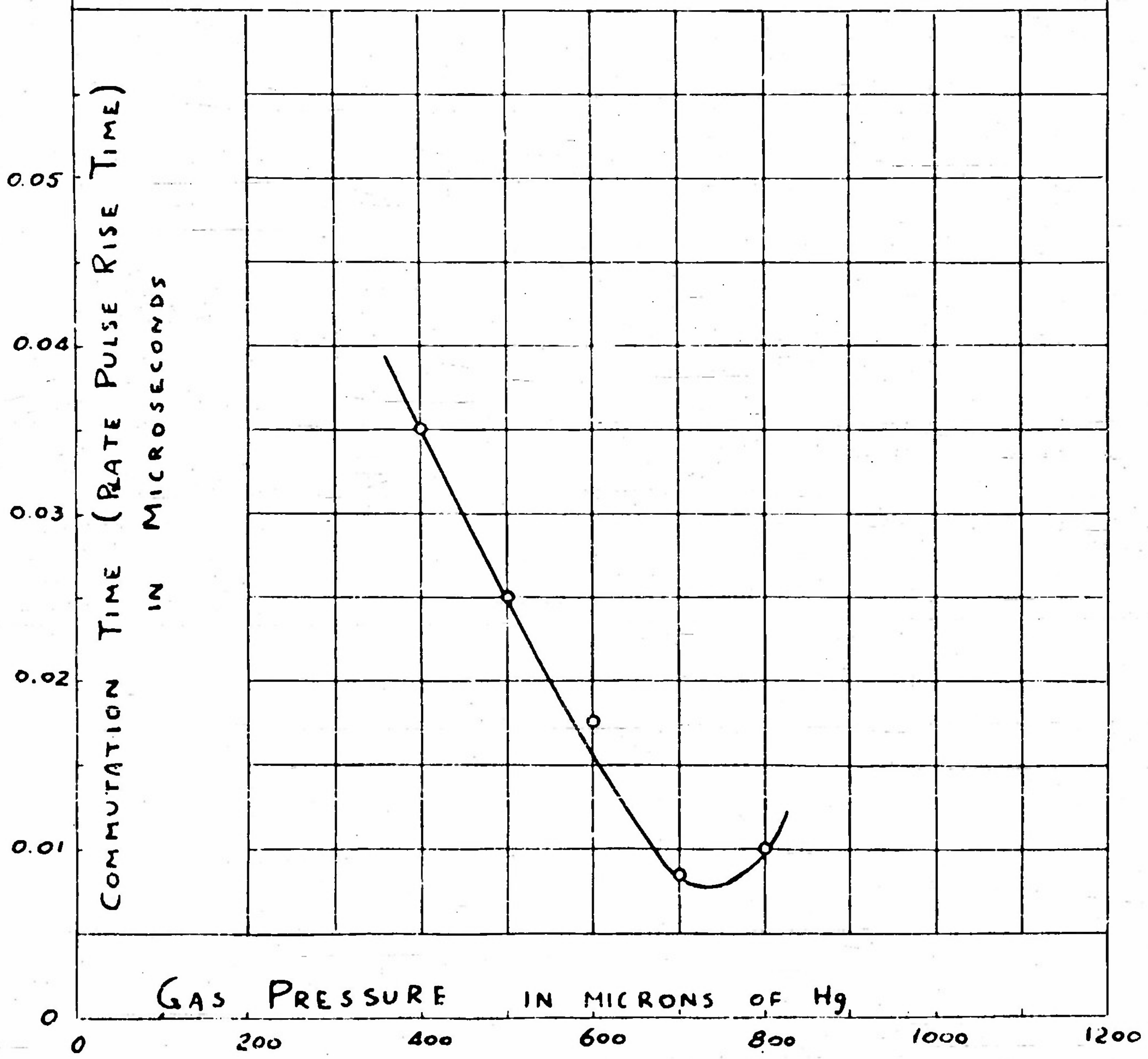
Voltage and pressure, however, cannot be increased together indefinitely. Eventually the forward voltage rating is exceeded and the tube will arc from plate to grid without trigger applied. The tube breakdown voltage $V_b$ is a function F of the pressure times the electrode separation (pd) as shown by the graph 4.[14]

$$V_b = F(pd)$$

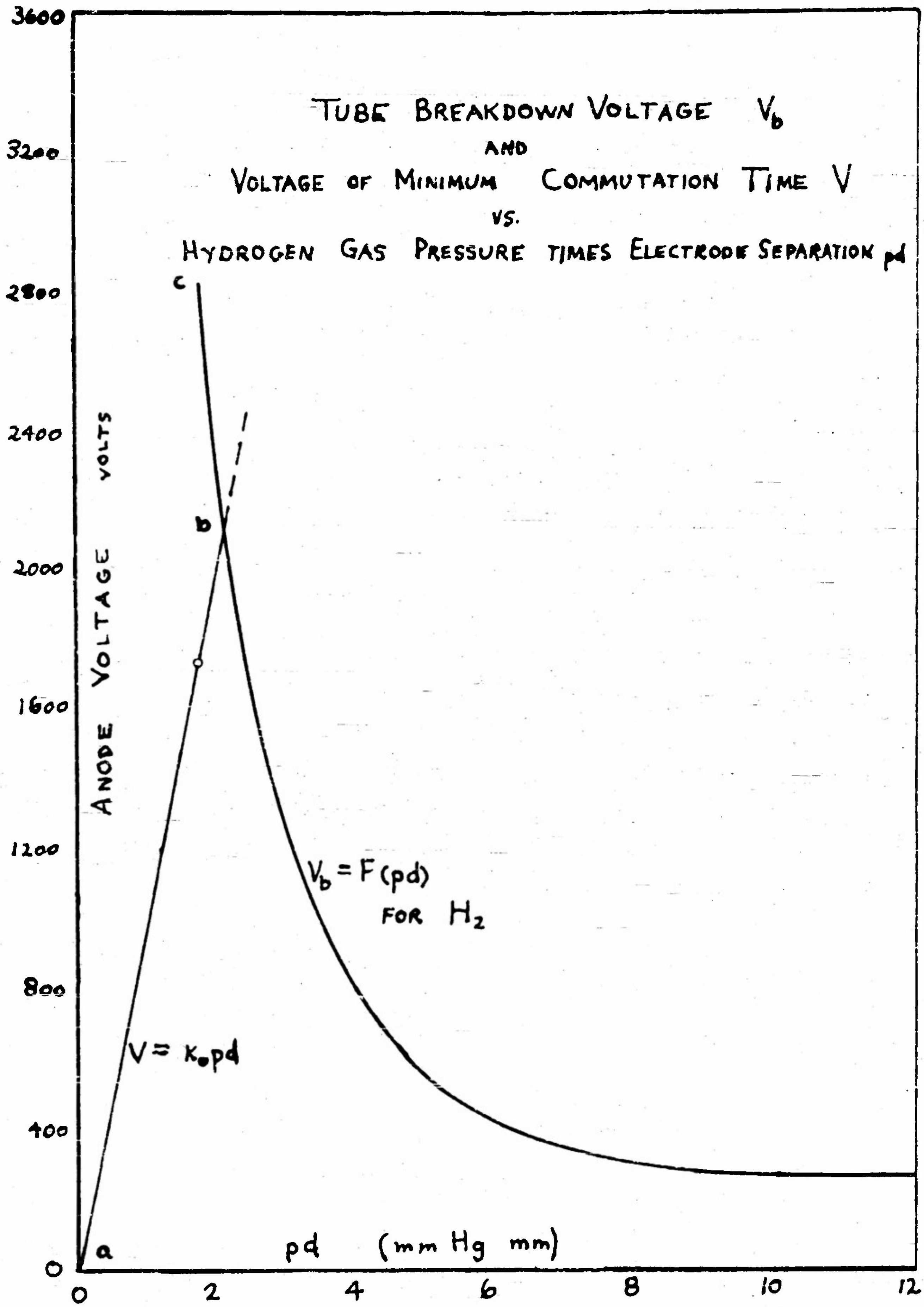
Therefore, the conditions for the minimum commutation time of a hydrogen thyratron at any given voltage will exist when $V = V_b = F\ (pd)$. Hence, for the normally high rated voltages of these hydrogen thyratrons, the commutation time will decrease as pressure increases up to the operational limits of the tube.

This relationship for the lowest commutation time is illustrated in Graph 4. In the region of anode voltages below about 2100 volts, the optimum operating parameter (pd) lies on the straight line <u>ab</u> where a ture minimum exists. For voltages above 2100 volts, use of this optimum condition would result in breakdown of the gas independent of grid voltage, so that

PLATE PULSE
COMMUTATION TIME
VS
GAS PRESSURE
FOR A
5C22 HYDROGEN THYRATRON




GRAPH 3.


TUBE BREAKDOWN VOLTAGE $V_b$
AND
VOLTAGE OF MINIMUM COMMUTATION TIME $V$
VS.
HYDROGEN GAS PRESSURE TIMES ELECTRODE SEPARATION $pd$
ANODE VOLTAGE VOLTS
3600
3200
2800
2400
2000
1600
1200
800
400
0
0
2
4
6
8
10
12
pd (mm Hg mm)
a
b
c
$V = K_0 pd$
$V_b = F(pd)$
FOR $H_2$

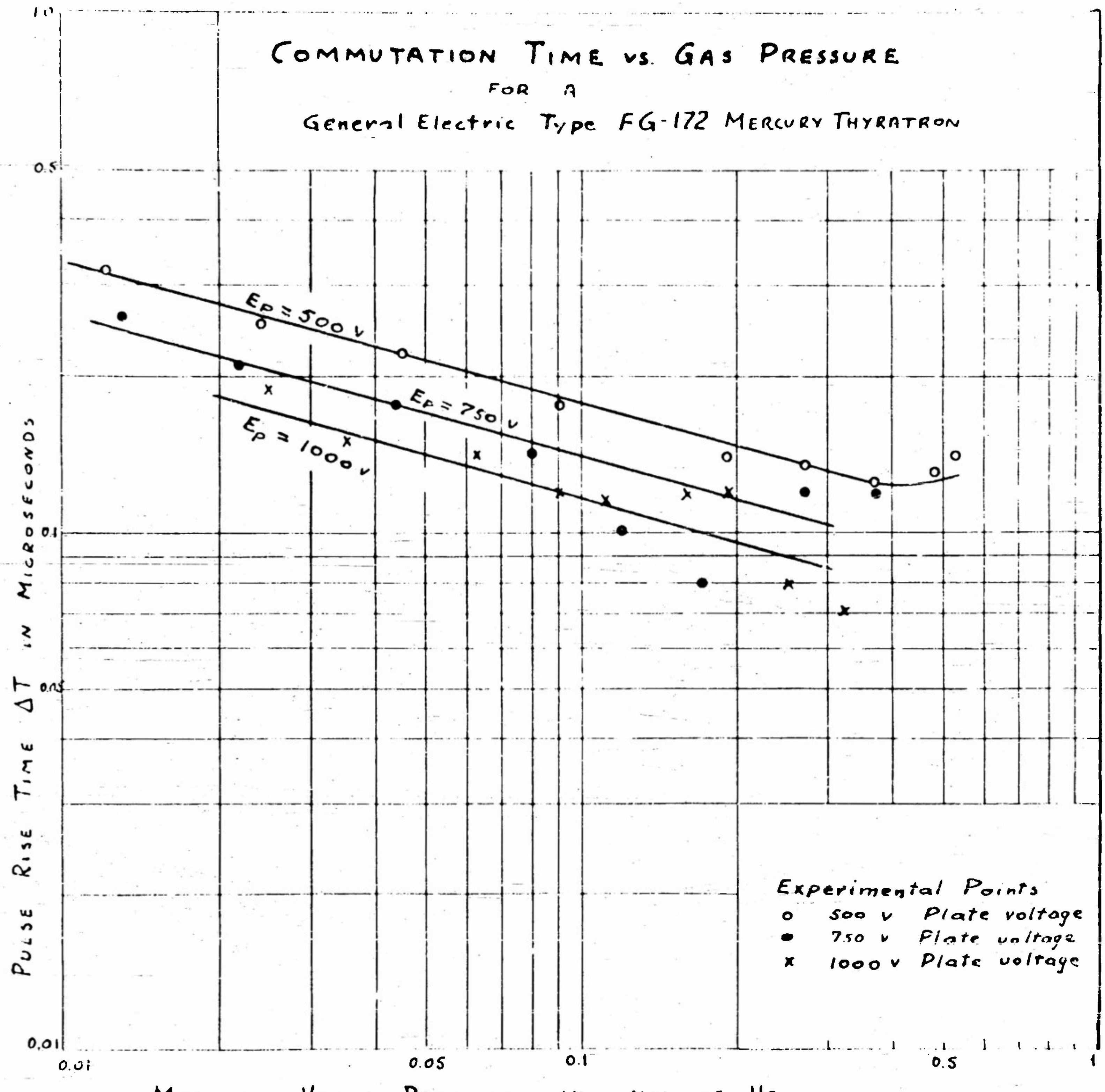
GRAPH 4.

the shortest commutation time is achieved by operating as near to the breakdown curve bg as safety considerations will allow.

These results have been verified experimentally. Graph 3 shows that a minimum commutation time for the 5C22 exists at a pressure of 700 microns of mercury with an anode potential of 1750 volts, but the tube will not support 2500 volts on the anode at the same pressure. Since the pressure must rise proportionally with the field, the minimum commutation time cannot be obtained at the higher voltage.

Similar results shown on graph 5 give the commutation time versus pressure for a mercury thyratron. The mercury vapor pressure was determined from measurements of the liquid mercury temperature since the liquid mercury and the mercury vapor are in equilibrium. This pressure-temperature relation is well known.[15] Even with a thermocouple soldered to the shell of the mercury thyratron nearest the liquid mercury pool as determined from an x-ray photograph of the tube, temperature and hence pressure measurements are inaccurate. Nevertheless, the curves show clearly the trend of decreasing commutation time with increasing pressure and a minimum commutation time at a plate potential of 500 volts.

This development shows that pressure and electrode separation are not independent variables, but that changes in gas pressure have the same effects toward minimizing the commutation time as changes in electrode separation in the highly-shielded hydrogen thyratrons with a given anode voltage applied.



COMMUTATION TIME vs. GAS PRESSURE
FOR A
General Electric Type FG-172 MERCURY THYRATRON
$E_P = 500$ v
$E_P = 750$ v
$E_P = 1000$ v
Experimental Points
o 500 v Plate voltage
● 750 v Plate voltage
x 1000 v Plate voltage
PULSE RISE TIME ΔT IN MICROSECONDS
MERCURY VAPOR PRESSURE IN MM OF Hg
1.0
0.5
0.1
0.05
0.01
0.01
0.05
0.1
0.5
1


GRAPH 5.

## SUMMARY

This thesis has shown the advantage of using a highly-shielded type of construction to keep plate current from flowing until a cathode-grid plasma has been established to minimize the commutation time of thyratrons. In this type of construction the commutation time and the tube breakdown voltage are both functions of the product of the gas pressure in the tube and the plate-grid separation. Effects of changes in one of these variables can be predicted by the known effects of changing the other one. Increasing gas pressure increases the number of gas molecules to be ionized, but decreases the energy electrons have at collisions by reducing their free paths. These opposing effects result in a maximum ionization rate between the low and high pressure extremes. This maximum ionization rate is noted by a minimum delay time in the grid circuit, but for the fields normally encountered in the plate-grid region of thyratrons, the forward anode voltage rating is exceeded before the conditions for the maximum ionization rate can be obtained. Hence, the commutation time normally decreases with increasing pressure.

WORK REMAINING TO BE DONE

Evidence that commutation times of hydrogen thyratrons are shorter than those of argon thyratrons which in turn are shorter than those of mercury thyratrons has been noted in a variety of tubes with widely differing geometries. Along with this experimental evidence is the fact that with a given energy an electron has a greater ionizing ability in argon and mercury than it does in hydrogen at the same pressure.[11] It is recommended that the effects of ionization rate and positive ion mobility upon the ionization processes in thyratrons be investigated for several different gases. It would be desirable to eliminate geometry and pressure effects by having several identical thyratrons of the highly-shielded type filled with several different gases at the same pressure.

MATHEMATICAL APPENDIX

The ionization rate is expressed as

$$C = \frac{A\rho^2}{\ell p_0} \int_{\bar{X}}^{D} (x - \bar{X})\, x^{1/2}\, \epsilon^{-\frac{x p}{\ell p_0}}\, dx$$

where $A = k E^{3/2} \left( \frac{q}{2m} \right)^{1/2}$ , a constant.

$$\int_{\bar{X}}^{D} x^{3/2} \epsilon^{-\frac{xp}{\ell p_0}} dx = \left[ -\frac{\ell p_0}{p} x^{3/2} \epsilon^{-\frac{xp}{\ell p_0}} \right]_{\bar{X}}^{D} + \frac{3}{2} \frac{\ell p_0}{p} \int_{\bar{X}}^{D} x^{1/2} \epsilon^{-\frac{xp}{\ell p_0}} dx$$

The integral $\int x^{1/2} \epsilon^{-\frac{xp}{\ell p_0}} dx$ can be expressed by using a series expansion for $\epsilon^{-\frac{xp}{\ell p_0}}$ and integrated term by term.

$$\epsilon^{-\frac{xp}{\ell p_0}} = 1 - \frac{1}{1!}\left(\frac{x\rho}{\ell p_0}\right) + \frac{1}{2!}\left(\frac{x\rho}{\ell p_0}\right)^2 - \frac{1}{3!}\left(\frac{x p}{\ell p_0}\right)^3 + \frac{1}{4!}\left(\frac{x p}{\ell p_0}\right)^4 - \text{error}$$

Because the series is alternating, the error is less than the last term.

$$x^{1/2} \epsilon^{-\frac{xp}{\ell p_0}} \cong x^{1/2} - \frac{1}{1!}\left(\frac{\rho}{\ell p_0}\right) x^{3/2} + \frac{1}{2!}\left(\frac{\rho}{\ell p_0}\right)^2 x^{5/2} - \frac{1}{3!}\left(\frac{\rho}{\ell p_0}\right)^3 x^{7/2} + \frac{1}{4!}\left(\frac{\rho}{\ell p_0}\right)^4 x^{9/2}$$

$$\int_{\bar{X}}^{D} x^{1/2} \epsilon^{-\frac{xp}{\ell p_0}} dx \cong \left[ \frac{2x^{3/2}}{3} - \frac{1}{1!}\left(\frac{\rho}{\ell p_0}\right) \frac{2x^{5/2}}{5} + \frac{1}{2!}\left(\frac{\rho}{\ell p_0}\right)^2 \frac{2x^{7/2}}{7} - \frac{1}{3!}\left(\frac{\rho}{\ell p_0}\right)^3 \frac{2x^{9/2}}{9} + \frac{1}{4!}\left(\frac{\rho}{\ell p_0}\right)^4 \frac{2x^{11/2}}{11} \right]_{\bar{X}}^{D}$$

Sample calculation:

$p = 100$ microns of Hg.

$P_o = 1000$ microns of Hg.

$X = .135$ cm.

$D = 1$ cm.

The mean free path of electrons in gases can be calculated approximately by the relation $L = \frac{1}{N \pi b^2}$

where $N$ = the number of gas molecules per unit volume which is $6.02 \times 10^{23}$ molecules per 22.4 liters at standard temperature ($0^\circ$ C) and pressure (760 mm. of Hg.).

b = the diameter of the hydrogen molecule = $1.36 \times 10^{16}$ cm. At 1 mm of Hg.,

$$N = \frac{6.02 \times 10^{23}}{22.4} \frac{\text{molecules}}{\text{liter}} \times \frac{1 \text{ liter}}{10^3 \text{ cm}^3} \times \frac{1}{760} = 3.54 \times 10^{6} \frac{\text{molecules}}{\text{cm}^3}$$

$$L = \frac{1}{N \pi b^2} = \frac{1}{3.54 \times 10^{16}} \times \frac{1}{\pi} \times \frac{1}{(1.36 \times 10^{8})^2} = .0485 \text{ cm. at } 0^\circ\text{C}$$

The mean free path of an electron in a gas varies with the gas temperature so assuming a gas temperature of 500° C.

$$L \text{ at } 500^\circ\text{C} = L \text{ at } 0^\circ\text{C} \times \left( \frac{T_{gas} \,^\circ\text{K}}{273 \,^\circ\text{K}} \right) = .14 \text{ cm.}$$

The mean free path is a function of electron velocity.[13] With 100 volts on the grid L = .14 cm. as shown, but with 50 volts on the grid the mean free path is somewhat less, assumed L = .08 cm. There was no way to obtain accurate measurements of some quantities such as hydrogen gas temperature which has considerable effect on the mean free path L. However, the actual value of the mean free path is not too important since the qualitative results of these calculations are fully illustrated by using a value in the correct order of magnitude.

$$\int_{.135}^{1} x^{1/2} \epsilon^{-\frac{x}{1.4}} dx \cong \left[ \frac{2}{3}(1) - \frac{1}{1.4}\left(\frac{2}{5}\right)(1) + \frac{1}{2}\left(\frac{1}{1.4}\right)^2 \frac{2}{7}(1) - \frac{1}{6}\left(\frac{1}{1.4}\right)^3 \frac{2}{9}(1) + \frac{1}{24}\left(\frac{1}{1.4}\right)^4 \frac{2}{11} \right]$$
$$- \left[ \frac{2}{3}(.05) - \frac{1}{1.4}\left(\frac{2}{5}\right)(.0067) + \frac{1}{2}\left(\frac{1}{1.4}\right)^2 \left(\frac{2}{7}\right)(.0009) \right]$$

$$\int_{.135}^{1} x^{1/2} \epsilon^{-\frac{x}{1.4}} dx \cong .667 - .286 + .073 - .013 + .002 - .033 + .002 = .410$$

$$\frac{e}{A} = \left(\frac{p}{l p_0}\right) p \left\{ \left[ - \frac{l p_0}{p} x^{3/2} \epsilon^{-\frac{x p}{l p_0}} \right]_{\bar{X}}^{D} + \left( \frac{3}{2} \frac{l p_0}{p} - \bar{X} \right) \int_{\bar{X}}^{D} x^{1/2} \epsilon^{-\frac{x p}{l p_0}} dx \right\}$$

$$\frac{C}{A} = \frac{1}{1.4}(100)\left[-1.4(1)(.492) + 1.4(.05)(.908) + \left(1.4(1.5) - \frac{.135}{1.4}\right).410\right]$$

$$\frac{C}{A} = 12.8$$

The integral can also be expressed in terms of the probability integral and evaluated directly. However, the values of some of the constants are not known too accurately. Hence, precise methods of evaluation are unwarranted when an easier approximation method can be used. The approximation used is the prismoidal formula.

$$\int_a^b f(x)dx = \frac{b-a}{6}\left[f(a) + 4f\left(\frac{b+a}{2}\right) + f(b)\right]$$

Sample calculation:

$$\frac{\rho}{\rho_0} = .1 \;, \quad D = 1 \text{ cm.} \;, \quad X = .135 \text{ cm.}$$

$$\int_X^D f(x)\,dx = \int_X^D (x-X)\,x^{1/2}\,\epsilon^{-\frac{x\rho}{X\rho_0}}\,dx = \frac{D-X}{6}\left[f(X) + 4f\left(\frac{D+X}{2}\right) + f(D)\right]$$

$$\int_X^D f(x)\,dx = \frac{1.-.135}{6}\left[0 + 4\left(\frac{1.135}{2} - .135\right)\left(\frac{1.135}{2}\right)^{1/2}\epsilon^{-\frac{1.135}{2\times 1.4}} + \right.$$
$$\left. + .865(1)\,\epsilon^{-\frac{1}{1.4}}\right]$$

$$\int_X^D f(x)\,dx = .186$$

$$\frac{C}{A} = P\left(\frac{\rho}{\rho_0}\right)\int_X^D f(x)\,dx = 100(.714)(.186) = 13.3$$

TABLE OF CALCULATIONS

$$f(x) = (x - \bar{X})\, x^{1/2}\, \epsilon^{-\frac{x}{\bar{X} p_0}}$$

| Pressure in microns of Hg. | $4\ f\left(\frac{\bar{X} + D}{2}\right)$ | f(D) | $\int_{\bar{X}}^{D} f(x)\,dx$ | C/A |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 100 | .870 | .425 | .186 | 13.3 |
| 200 | .705 | .208 | .132 | 37.8 |
| 300 | .410 | .102 | .0738 | 47.4 |
| 400 | .257 | .0493 | .044 | 50.5 |
| 500 | .174 | .024 | .0285 | 50.8 |
| 600 | .113 | .012 | .018 | 47.5 |
| 700 | .0757 | .0058 | .0117 | 41.0 |
| 800 | .051 | .0028 | .00775 | 35.5 |
| 900 | .034 | .00135 | .0051 | 29.5 |
| 1000 | .0225 | .00067 | .00334 | 24.0 |

The second table of calculations is for a trigger voltage of 50 volts instead of 100 volts on the grid. The constants changed are:

$$\bar{X} = \frac{V_i}{E} = 2 \times .135 = .27 \text{ cm.}$$

$$E = \tfrac{1}{2} \times 100 = 50 \text{ v/cm.}$$

$$\ell = .08 \text{ cm.}$$

| Pressure in microns of Hg. | $4\ f\left(\frac{\bar{X} + D}{2}\right)$ | f(D) | $\int_{\bar{X}}^{D} f(x)\,dx$ | C/A |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 100 | .543 | .219 | .0928 | 11.1 |
| 200 | .254 | .0656 | .0390 | 18.7 |
| 300 | .131 | .0197 | .0185 | 20.0 |
| 400 | .0546 | .0060 | .0074 | 14.2 |
| 500 | .0256 | .00185 | .00334 | 10.0 |
| 600 | .0148 | .00055 | .00162 | 7.0 |
| 700 | .0056 | .0017 | .00070 | 4.1 |
| 800 | .00256 | .00005 | .00032 | 2.4 |
| 900 | .0012 | .00001 | .00015 | 1.4 |

The curves showing these data of ionization rate versus pressure for two different trigger voltages are shown on graph 2.

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